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# Constraints on four-Fermi contact interactions from low-energy electroweak experiments

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**Abstract.** We investigate the constraints on four-Fermi contact interactions from low-energy lepton-quark and lepton-lepton scattering experiments — polarization asymmetries in electron (muon)-nucleon scattering experiments, cesium and thallium atom parity violation measurements, neutrino-nuclei and neutrino-electron scattering experiments. These constraints are then combined by assuming the lepton and quark universalities and $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge invariance of the contact interaction, which leave independent six lepton-quark and three pure-leptonic interactions. Impacts of these constraints on models with an additional $Z$-boson are briefly discussed. We also present updates of the low-energy constraints on the $S$ and $T$ parameters.

## 1 Introduction

Searching for an evidence of physics beyond the Standard Model (SM) is one of the most important subjects in the field of particle physics. Up to now, no particles other than the SM particles have been found at collider experiments. This fact implies that the mass scale of new particles should be larger than several hundred GeV.

Low-energy neutral current (L.E.N.C.) phenomena is mediated by exchange of the photon and the $Z$-boson in the SM. We can parametrize new physics contributions to L.E.N.C. phenomenon as effective four-Fermi contact interactions.

Generally, the effective contact interactions for neutral currents among quarks and leptons can be parametrized as

$$\mathcal{L}_{NC} = \sum_{f,f'} \sum_{\alpha,\beta} \eta^{ff'}_{\alpha\beta} \; \overline{\psi_f} \gamma^\mu P_\alpha \psi_f \; \overline{\psi_{f'}} \gamma_\mu P_\beta \psi_{f'} \ , \qquad (1.1)$$

where $f, f'$ stand for lepton and quarks, $\alpha, \beta = L, R$ denote their chirality: $P_{L(R)} = (1 - (+)\gamma_5)/2$. The coefficients $\eta^{\ell q}_{\alpha\beta}$ have the dimension of $(\mathrm{mass})^{-2}$ which is often expressed as [1,2],

$$\eta^{ff'}_{\alpha\beta} = \frac{4\pi}{(\Lambda^{ff'+}_{\alpha\beta})^2} \quad \text{or} \quad -\frac{4\pi}{(\Lambda^{ff'-}_{\alpha\beta})^2} \ . \qquad (1.2)$$

The experimental limits of the scale $\Lambda$ are given for some combinations of $(f, f')$, $(\alpha, \beta)$ and the overall sign factor $(+,-)$ [2]. If the contact interactions are results of an exchange of an extra heavy neutral vector boson $Z_E$, they are given by

$$\eta^{ff'}_{\alpha\beta} = -\frac{g^f_\alpha g^{f'}_\beta}{m^2_{Z_E}}, \qquad (1.3)$$

where $g^f_\alpha$ and $g^{f'}_\beta$ are the $Z_E$-boson couplings to $f_\alpha$ and $f'_\beta$, respectively. An exchange of a heavy boson in the "$s$-channel" of the processes $ff' \longrightarrow ff'$ or $f\overline{f'} \longrightarrow f\overline{f'}$ can also be expressed in the form of (1.1) as long as we ignore the contribution which does not interfere with the leading SM amplitudes.

In this report, we present constraints from low-energy electroweak measurements on the coefficients $\eta^{ff'}_{\alpha\beta}$, in order to take advantage of the model-independent nature of the parametrization (1.1). As a simple example, the constraints on the $\eta^{ff'}_{\alpha\beta}$ terms are used to derive constraints on an extra $Z$ boson parameters.

Recently there have been renewed interest in the possible existence of new interactions between quarks and leptons because of an excess of events at high $Q^2$ $e^+p$ inelastic scattering at HERA, which was reported by H1 [3] and ZEUS [4] collaborations. Such an excess of events may be reproduced by introducing some lepton-quark contact interactions [5]. Therefore, it is worth examining constraints on those contact interactions from all low-energy electroweak measurements as model-independent as possible.

We study in this report the following four types of low-energy electroweak experiments; (i) polarization asymmetry of the charged lepton scattering off nucleus target, (ii) parity violation in cesium and thallium atoms, (iii) inelastic $\nu_\mu$ scattering off nucleus target and (iv) $\nu_\mu(\bar{\nu}_\mu)$

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– electron scattering experiments. Individual experiment receives contributions of a different combination of the contact interactions. We therefore present constraints on the contact terms from each experiment separately. These constraints are then combined by assuming that the contact interactions satisfy the flavor universality and the $SU(2)_L \times U(1)_Y$ gauge invariance of the SM.

This paper is organized as follows. In the next section, we review the approach of [6] that we adopt for calculating the low-energy electroweak observables with the model-independent framework of [7]. Section 3 is devoted to survey the experimental data corresponding to the above four types of experiments. The SM predictions are given both as functions of $m_t$ and $m_H$ in the minimal SM and as a function of $S$ and $T$ [8] in generic $SU(2)_L \times U(1)_Y$ model. We adopt the slightly modified version of the $S$ and $T$ variables that was introduced in [6]. In Sect. 4 we present individual constraints on the $S$ and $T$ parameters in the generic $SU(2)_L \times U(1)_Y$ model, and compare the low-energy constraints with the constraints from the $Z$-pole experiments [9]. They update the results of [6,10,9]. In Sect. 5 we obtain constraints on the contact interactions by assuming no new physics contributions to the $S$ and $T$ parameters. In Sect. 6, we present constraints on models with an extra $Z$-boson as an example of the use of our model-independent parametrization of the low-energy data in terms of the four-fermi contact interactions.

## 2 Framework

The effective Lagrangian for the lepton-quark four-Fermi interaction is given as follows;

$$\begin{aligned}\mathcal{L}^{\ell q}_{eff} = -\frac{G_F}{\sqrt{2}} \sum_{q=u,d} \Big[ & C_{1q}\, \overline{\psi_\ell}\gamma^\mu\gamma_5\psi_\ell\, \overline{\psi_q}\gamma_\mu\psi_q \\ & + C_{2q}\, \overline{\psi_\ell}\gamma^\mu\psi_\ell\, \overline{\psi_q}\gamma_\mu\gamma_5\psi_q \\ & + C_{3q}\, \overline{\psi_\ell}\gamma^\mu\gamma_5\psi_\ell\, \overline{\psi_q}\gamma_\mu\gamma_5\psi_q \Big], \end{aligned} \quad (2.1)$$

where $\ell = e, \mu, \tau$ and $q = u, d$. Adding to the model-independent parameters $C_{1q}$ and $C_{2q}$ of [7], we introduce the parameter $C_{3q}$ as the coefficient of the axial vector-axial vector current. They can be expressed in terms of the helicity amplitude $M^{\ell q}_{\alpha\beta}$ of [6] as

$$C_{1q} = \frac{1}{2\sqrt{2}G_F}\Big( M^{\ell q}_{LL} + M^{\ell q}_{LR} - M^{\ell q}_{RL} - M^{\ell q}_{RR}\Big), \quad (2.2a)$$

$$C_{2q} = \frac{1}{2\sqrt{2}G_F}\Big( M^{\ell q}_{LL} - M^{\ell q}_{LR} + M^{\ell q}_{RL} - M^{\ell q}_{RR}\Big), \quad (2.2b)$$

$$C_{3q} = \frac{1}{2\sqrt{2}G_F}\Big( -M^{\ell q}_{LL} + M^{\ell q}_{LR} + M^{\ell q}_{RL} - M^{\ell q}_{RR}\Big). \quad (2.2c)$$

With the above identification, $q^2$-dependence of higher order electroweak effects are properly taken into account by making use of the effective Lagrangian formalism. Introducing the effective form factors $\bar{e}^2(q^2)$, $\bar{g}^2_Z(q^2)$, $\bar{s}^2(q^2)$, the helicity amplitude $M^{ff'}_{\alpha\beta}$ of the neutral current process $f_\alpha f'_\beta \longleftrightarrow f_\alpha f'_\beta$ can be expressed as [6];

$$\begin{aligned} M^{ff'}_{\alpha\beta}(q^2)^{\text{‘SM’}} = & \frac{1}{q^2}\Big\{ (Q_f\, Q_{f'})\left[\bar{e}^2(q^2) + \hat{e}^2\, \Gamma^f_1(q^2)\right. \\ & \left. + \hat{e}^2\, \Gamma^{f'}_1(q^2)\right] + (Q_f\, I_{3f'})\, \hat{e}^2\, \overline{\Gamma}^{f'}_2(q^2) \\ & + (Q_{f'}\, I_{3f})\, \hat{e}^2\, \overline{\Gamma}^{f}_2(q^2)\Big\} \\ & + \frac{1}{q^2 - m_Z^2}\Big\{ (I_{3f} - Q_f\hat{s}^2) \\ & \times (I_{3f'} - Q_{f'}\hat{s}^2)\bar{g}^2_Z(0) \\ & - (I_{3f} - Q_f\hat{s}^2)\, Q_{f'}\, \hat{g}^2_Z\left[\bar{s}^2(q^2) - \hat{s}^2\right] \\ & - (I_{3f'} - Q_{f'}\hat{s}^2)\, Q_f\, \hat{g}^2_Z\left[\bar{s}^2(q^2) - \hat{s}^2\right]\Big\} \\ & + B^{NC}_{ff'}(0,0) + O\Big(\hat{e}^2\frac{q^2}{m_W^2}\Big), \end{aligned} \quad (2.3)$$

in the generic $SU(2)_L \times U(1)_Y$ model, where $\hat{e}, \hat{g}_Z$ are the $\overline{\text{MS}}$ couplings that satisfy $\hat{e} = \hat{g}\hat{s} = \hat{g}_Z\hat{s}\hat{c}$ which are renormalized as $\hat{e}^2 = \bar{e}^2(m_Z^2)$ and $\hat{s}^2 = 1 - \hat{c}^2 = \bar{s}^2(m_Z^2)$. $Q_f, I_{3f}(f = \ell, q)$ denote the electric charge and the third component of weak isospin, respectively, of the corresponding fermion $f_\alpha$. $\Gamma^f_i, \overline{\Gamma}^f_i (i = 1, 2)$ and $B^{NC}_{ff'}$ stand for the vertex and box corrections, respectively, and their explicit forms are given in Appendix A of [6]. In the presence of the contact interactions, the complete helicity amplitude is given by the sum of (2.3) and the contact term;

$$M^{ff'}_{\alpha\beta}(q^2) = M^{ff'}_{\alpha\beta}(q^2)^{\text{‘SM’}} + \eta^{ff'}_{\alpha\beta}. \quad (2.4)$$

Accordingly the coefficient $C_{iq}$ of the effective lepton-quark interactions can be divided into two pieces as,

$$C_{iq} = C^{\text{‘SM’}}_{iq} + \Delta C_{iq}, \quad (2.5)$$

where the first term denotes the contribution with the generic $SU(2)_L \times U(1)_Y$ model, while the second term arises from the contact interactions. Hereafter, we denote by ‘SM’ the predictions of the generic $SU(2)_L \times U(1)_Y$ model, where the form factors $\bar{g}^2_Z(m_Z^2)$ and $\bar{s}^2(m_Z^2)$, or $S$ and $T$ parameters, are treated as free parameters. In the minimal SM, they are determined by $m_t$ and $m_H$, whose explicit forms are given below [6,9]. The coefficients $C^{\text{‘SM’}}_{iq}$ are approximately expressed as

$$\begin{aligned} C^{\text{‘SM’}}_{1q} = & \; I_{3q} - 2Q_q \sin^2\theta_W \\ & + \text{ higher order terms}, \end{aligned} \quad (2.6a)$$

$$\begin{aligned} C^{\text{‘SM’}}_{2q} = & \; I_{3q}(1 - 4\sin^2\theta_W) \\ & + \text{ higher order terms}, \end{aligned} \quad (2.6b)$$

$$C^{\text{‘SM’}}_{3q} = -I_{3q} \quad + \text{ higher order terms}. \quad (2.6c)$$

The terms $\Delta C_{iq}$ receive contributions from the contact interactions,

$$\Delta C_{1q} = \frac{1}{2\sqrt{2}G_F}\Big( \eta^{\ell q}_{LL} + \eta^{\ell q}_{LR} - \eta^{\ell q}_{RL} - \eta^{\ell q}_{RR}\Big), \quad (2.7a)$$

$$\Delta C_{2q} = \frac{1}{2\sqrt{2}G_F}\left( \eta_{LL}^{\ell q} - \eta_{LR}^{\ell q} + \eta_{RL}^{\ell q} - \eta_{RR}^{\ell q}\right), \quad (2.7b)$$

$$\Delta C_{3q} = \frac{1}{2\sqrt{2}G_F}\left( -\eta_{LL}^{\ell q} + \eta_{LR}^{\ell q} + \eta_{RL}^{\ell q} - \eta_{RR}^{\ell q}\right). \quad (2.7c)$$

For neutrino-quark scattering, it is conventional to introduce the model independent parameters, $g_\alpha^2$ and $\delta_\alpha^2$ $(\alpha = L, R)$ [24];

$$g_\alpha^2 \equiv u_\alpha^2 + d_\alpha^2, \quad (2.8a)$$
$$\delta_\alpha^2 \equiv u_\alpha^2 - d_\alpha^2, \quad (2.8b)$$

where, $u_\alpha$ and $d_\alpha$ are given by using the helicity amplitude as [6],

$$q_\alpha = -\frac{1}{2\sqrt{2}G_F} M_{L\alpha}^{\nu_\mu q} \quad (2.9a)$$
$$= q_\alpha^{\text{'SM'}} + \Delta q_\alpha \quad (q = u, d), \quad (2.9b)$$

The 'SM' contributions are obtained from [6,9] and the contribution from the contact interactions is

$$\Delta q_\alpha = -\frac{1}{2\sqrt{2}G_F}\eta_{L\alpha}^{\nu_\mu q}. \quad (2.10)$$

For the neutrino-electron scattering experiments, we use the experimental data of the total cross sections for $\nu_\mu$-$e$ and $\bar{\nu}_\mu$-$e$ scatterings, which are expressed in terms of the helicity amplitude as [6];

$$\frac{\sigma^{\nu e}}{E_\nu} = \frac{m_e}{4\pi}\left\{ \left| M_{LL}^{\nu_\mu e}(\langle Q^2\rangle = m_e E_\nu)\right|^2 + \frac{1}{3}\left| M_{LR}^{\nu_\mu e}(\langle Q^2\rangle = \frac{m_e E_\nu}{2})\right|^2 \right\}, \quad (2.11a)$$

$$\frac{\sigma^{\bar{\nu} e}}{E_{\bar{\nu}}} = \frac{m_e}{4\pi}\left\{ \frac{1}{3}\left| M_{LL}^{\nu_\mu e}(\langle Q^2\rangle = \frac{m_e E_{\bar{\nu}}}{2})\right|^2 + \left| M_{LR}^{\nu_\mu e}(\langle Q^2\rangle = m_e E_{\bar{\nu}})\right|^2 \right\}. \quad (2.11b)$$

Here we replace the integral over the momentum transfer $Q^2$ by $m_e E_\nu$. The amplitudes are then parametrized as

$$M_{L\alpha}^{\nu_\mu e} = (M_{L\alpha}^{\nu_\mu e})^{\text{'SM'}} + \eta_{L\alpha}^{\nu_\mu e}. \quad (2.12)$$

Next, we briefly review how we estimate the contribution of generic $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ model. In the formalism of [6], the L.E.N.C. observables are expressed in terms of two form factors, $\bar{g}_Z^2(0)$ and $\bar{s}^2(0)$. They can be expressed by the $S$ and $T$-parameters [8] as follows;

$$\bar{g}_Z^2(0) \approx 0.5456 + 0.0040T, \quad (2.13a)$$
$$\bar{s}^2(0) \approx 0.2418 + 0.0034S'' - 0.0023T, \quad (2.13b)$$

where $S''$ is defined by

$$S'' \equiv S - 1.30\delta_\alpha. \quad (2.14)$$

The parameter $\delta_\alpha \equiv 1/\bar{\alpha}(m_Z^2) - 128.72$ was introduced in [6] to take care of the uncertainty in $\bar{\alpha}(m_Z^2)$. The recent estimation by Eidelman and Jegerlehner [11] gives $\delta_\alpha = 0.03 \pm 0.09$ [9]. In the minimal SM, the $S$ and $T$-parameters are parametrized by $m_t$ and $m_H$ as [9],

$$S_{\text{SM}} \approx -0.233 - 0.007x_t + 0.091x_H - 0.010x_H^2, \quad (2.15a)$$
$$T_{\text{SM}} \approx 0.879 + (0.130 - 0.003x_H)x_t + 0.003x_t^2 - 0.079x_H - 0.028x_H^2 + 0.0026x_H^3, \quad (2.15b)$$

where $x_t$ and $x_H$ are

$$x_t \equiv (m_t - 175\ \text{GeV})/10\ \text{GeV}, \quad (2.16a)$$
$$x_H \equiv \log(m_H/100\ \text{GeV}), \quad (2.16b)$$

respectively[1]. It is convenient to introduce the following parameters;

$$\Delta S \equiv S - S_{SM}^0 = S + 0.233, \quad (2.17a)$$
$$\Delta T \equiv T - T_{SM}^0 = T - 0.879, \quad (2.17b)$$

where $S_{\text{SM}}^0$ and $T_{\text{SM}}^0$ are the SM predictions for $S$ and $T$ at $m_t = 175$ GeV, $m_H = 100$ GeV.

## 3 Low-energy electroweak observables

### 3.1 Polarization asymmetry of charged lepton-nucleus scattering

In this subsection, we study four experiments on charged lepton-nucleus scattering, in which two types of observables were measured. First, polarization asymmetry $A$ of charged lepton scattering off nucleus target

$$A \equiv \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}, \quad (3.1)$$

has been measured in $e$D-scattering at SLAC [12], in $e$C scattering at Bates [13] and in $e$Be scattering at Mainz [14]. Here $d\sigma_{L(R)}$ denotes the differential cross section of the left- (right-) handed lepton scattering off target. These are Parity-odd asymmetries. Another type of the asymmetry is the polarization and charge asymmetry parameter $B$

$$B \equiv \frac{d\sigma_L^+ - d\sigma_R^-}{d\sigma_L^+ + d\sigma_R^-}, \quad (3.2)$$

where $d\sigma_{R(L)}^-$ is the differential cross section of right- (left-) handed negatively charged lepton scattering off nucleus target, wheres $d\sigma_{R(L)}^+$ denotes those of positively charged anti-lepton scattering. The parameter $B$ has been measured in $\mu^\pm$C scattering at CERN [15]. The measurement of the asymmetry parameter $A$ constrains a combination of $C_{1q}$ and $C_{2q}$, wheres the asymmetry $B$ constrains the parameter $C_{3q}$.

[1] These parametrizations are valid in the mass range 160 GeV $< m_t <$ 185 GeV and 40 GeV $< m_H <$ 1000 GeV

#### 3.1.1 SLAC $e$D experiment

The historic SLAC experiment [12] that established parity violation in the electron-quark neutral current scattering still gives non-trivial constraints on new physics contribution. Here we quote the results of the analysis of [6]. The parameter $A_{SLAC}$ is expressed in [6] as

$$A_{SLAC} = -\frac{6G_F Q^2}{5\sqrt{2}\bar{e}^2(-Q^2)}\left\{\left(2C_{1u} - C_{1d}\right)\left(1 - \frac{3}{4}c\right) + \left(2C_{2u} - C_{2d}\right)\left(b + \frac{5}{12}c\right)\right\}, \quad (3.3)$$

where the terms $b$ and $c$ represent deviations from the valence-quark approximation. After studying the uncertainties in the correction terms $b$ and $c$, the following constraints were obtained [6] for the model-independent parameters

$$\left.\begin{aligned} 2C_{1u} - C_{1d} &= \phantom{-}0.94 \pm 0.26 \\ 2C_{2u} - C_{2d} &= -0.66 \pm 1.23 \end{aligned}\right\} \rho_{\mathrm{corr}} = -0.975. \quad (3.4)$$

The theoretical prediction within the generic $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ model at $\langle Q^2 \rangle \simeq 1.5\ \mathrm{GeV}^2$ is given as;

$$(2C_{1u} - C_{1d})^{\text{'SM'}} \approx 0.723 - 0.0115\Delta S + 0.0129\Delta T, \quad (3.5a)$$

$$(2C_{2u} - C_{2d})^{\text{'SM'}} \approx 0.105 - 0.0207\Delta S + 0.0144\Delta T. \quad (3.5b)$$

The minimal SM predicts

$$(2C_{1u} - C_{1d})^{\mathrm{SM}} \approx 0.723 + 0.002x_t - 0.0024x_H, \quad (3.6a)$$

$$(2C_{2u} - C_{2d})^{\mathrm{SM}} \approx 0.105 + 0.002x_t - 0.0032x_H. \quad (3.6b)$$

#### 3.1.2 CERN $\mu^{\pm}$C experiment

Charge and polarization asymmetry of $\mu^{\pm}$ deep inelastic scattering off $^{12}$C target has been measured at CERN [15]. In the parton model, the asymmetry parameter $B$ is given as

$$B = -\frac{6G_F}{5\sqrt{2}\bar{e}^2(-Q^2)}\frac{1-(1-y)^2}{1+(1-y)^2}Q^2\left[(-2C^{\mu}_{3u} + C^{\mu}_{3d}) + |\lambda|(-2C^{\mu}_{2u} + C^{\mu}_{2d})\right]\frac{q(x) - \bar{q}(x)}{D(x)}, \quad (3.7)$$

where $\lambda$ denotes the effective $\mu^{\pm}$ beam polarization. Here we assumed that charm and strange quarks have the same effective interaction as up and down quarks, respectively: $C^{\mu}_{1c} = C^{\mu}_{1u}$ and $C^{\mu}_{1s} = C^{\mu}_{1d}$. The superscript $\mu$ is put to remind us that these parameters are for the $\mu$-$q$ scattering. All the other experiments are done for electron scatterings, and the corresponding superscript $e$ is suppressed. $q(x), \bar{q}(x)$ and $D(x)$ are expressed in terms of the quark and anti-quark distribution functions in the nucleus as

$$q(x) = u(x) + d(x) + s(x) + c(x), \quad (3.8a)$$

$$\bar{q}(x) = \bar{u}(x) + \bar{d}(x) + \bar{s}(x) + \bar{c}(x), \quad (3.8b)$$

$$D = u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + \frac{2}{5}(s(x) + \bar{s}(x)) + \frac{8}{5}(c(x) + \bar{c}(x)). \quad (3.8c)$$

The experiment has been performed at two different beam energies, $E = 200$ GeV and 120 GeV. The mean momentum transfer of the experiments may be estimated as $\langle Q^2 \rangle \simeq 50\ \mathrm{GeV}^2$ [16]. The experimental data give the following constraints on the coefficients of $C_{2q}$ and $C_{3q}$;
(i) $E = 200$ GeV:

$$\begin{aligned} &(2C_{3u} - C_{3d}) + |\lambda|(2C_{2u} - C_{2d}) = -1.51 \pm 0.43, \\ &|\lambda| = 0.81 \pm 0.04, \end{aligned} \quad (3.9)$$

(ii) $E = 120$ GeV:

$$\begin{aligned} &(2C_{3u} - C_{3d}) + |\lambda|(2C_{2u} - C_{2d}) = -1.79 \pm 0.83, \\ &|\lambda| = 0.66 \pm 0.05. \end{aligned} \quad (3.10)$$

Combining the above two constraints, we find

$$\left.\begin{aligned} 2C_{3u} - C_{3d} &= -3.01 \pm 4.92 \\ 2C_{2u} - C_{2d} &= \phantom{-}1.85 \pm 6.31 \end{aligned}\right\} \rho_{\mathrm{corr}} = -0.997. \quad (3.11)$$

The theoretical prediction of the generic $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ model at $\langle Q^2 \rangle \simeq 50\ \mathrm{GeV}^2$ is given as;

$$(2C_{3u} - C_{3d})^{\text{'SM'}} \approx -1.505 - 0.011\Delta T, \quad (3.12a)$$

$$(2C_{2u} - C_{2d})^{\text{'SM'}} \approx 0.109 - 0.021\Delta S + 0.015\Delta T. \quad (3.12b)$$

The minimal SM predicts,

$$(2C_{3u} - C_{3d})^{\mathrm{SM}} \approx -1.505 - 0.001x_t + 0.001x_H, \quad (3.13a)$$

$$(2C_{2u} - C_{2d})^{\mathrm{SM}} \approx 0.109 + 0.002x_t - 0.003x_H. \quad (3.13b)$$

The superscript $\mu$ has been suppressed since the predictions of the SM and the generic $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ model are essentially the same for $e$ and $\mu$. The difference between the predictions for $2C_{2u} - C_{2d}$ in (3.5) and (3.12) reflects different $\langle Q^2 \rangle$ of the two experiments.

#### 3.1.3 Bates $e$C experiment

The polarization asymmetry parameter $A$ of electron elastic scattering off $^{12}$C target has been measured at Bates [13]

$$A_{Bates} = -3\sqrt{2}\frac{G_F Q^2}{\bar{e}^2(-Q^2)}\left(C_{1u} + C_{1d}\right). \quad (3.14)$$

Here we neglected contribution from quarks other than the $u$ and $d$ quarks, which have been estimated to be small under the condition of the experiment [17] at a typical momentum transfer of $\langle Q^2\rangle = 0.0225$ GeV$^2$. From the experimental data

$$A_{Bates} = (1.62 \pm 0.38) \times 10^{-6}, \tag{3.15}$$

we find by using $4\pi/\bar{e}^2(-0.0225 \text{ GeV}^2) = 135.87$,

$$C_{1u} + C_{1d} = -0.137 \pm 0.033. \tag{3.16}$$

The corresponding theoretical prediction at $\langle Q^2\rangle = 0.0225$ GeV$^2$ is found to be

$$(C_{1u} + C_{1d})^{\text{‘SM’}} \approx -0.1522 - 0.0023\Delta S + 0.0004\Delta T, \tag{3.17}$$

in the generic $\text{SU}(2)_L \times \text{U}(1)_Y$ model, and

$$(C_{1u} + C_{1d})^{\text{SM}} \approx -0.1522 + 0.0001x_t - 0.0002x_H, \tag{3.18}$$

in the minimal SM.

### 3.1.4 Mainz $e$Be experiment

At Mainz, polarization asymmetry of electron quasi-elastic scattering off $^9$Be target has been measured [14]. The experiment was performed at the mean momentum transfer $\langle Q^2\rangle \simeq 0.2025$ GeV$^2$. In this experiment, the asymmetry parameter $A_{Mainz}$ can be expressed by the following combination of the model independent parameters [14];

$$A_{Mainz} = -2.73C_{1u} + 0.65C_{1d} - 2.19C_{2u} + 2.03C_{2d}, \tag{3.19}$$

and the experimental result is given by

$$A_{Mainz} = -0.94 \pm 0.19. \tag{3.20}$$

Here the error is obtained by taking the quadratic sum of the theoretical and experimental ones as quoted in [14].

The theoretical prediction in the generic $\text{SU}(2)_L \times \text{U}(1)_Y$ model is

$$A^{\text{‘SM’}}_{Mainz} \approx -0.875 + 0.043\Delta S - 0.035\Delta T, \tag{3.21}$$

and that of the minimal SM is,

$$A^{\text{SM}}_{Mainz} \approx -0.875 - 0.005x_t + 0.007x_H. \tag{3.22}$$

## 3.2 Atomic parity violation

The experimental results of parity violation in atoms are often given in terms of the weak charge $Q_W(A,Z)$ of the nuclei. Using the model-independent parameter $C_{1q}$, the weak charge of a nuclei $(A,Z)$ can be expressed as;

$$Q_W(A,Z) = 2ZC_{1p} + 2(A-Z)C_{1n}, \tag{3.23}$$

where two coefficients $C_{1p}$ and $C_{1n}$ are given in terms of $C_{1u}$ and $C_{1d}$ as [6]

$$C_{1p} \approx 2C_{1u} + C_{1d} + 0.01349, \tag{3.24a}$$
$$C_{1n} \approx C_{1u} + 2C_{1d} + 0.00669. \tag{3.24b}$$

The theoretical prediction for $C_{1u}$ and $C_{1d}$ is found to be

$$C^{\text{‘SM’}}_{1u} \approx \phantom{-}0.18185 - 0.0045\Delta S + 0.0043\Delta T, \tag{3.25a}$$
$$C^{\text{‘SM’}}_{1d} \approx -0.34116 + 0.0023\Delta S - 0.0040\Delta T, \tag{3.25b}$$

in the generic $\text{SU}(2)_L \times \text{U}(1)_Y$ model, and

$$C^{\text{SM}}_{1u} \approx \phantom{-}0.18185 + 0.00059x_t - 0.00073x_H, \tag{3.26a}$$
$$C^{\text{SM}}_{1d} \approx -0.34116 - 0.00054x_t + 0.00051x_H, \tag{3.26b}$$

in the minimal SM.

There are two accurate measurements of the weak charge in the cesium atom [18,19];

$$Q_W({}^{133}_{55}Cs) = \begin{cases} -71.04 \pm 1.58 \pm 0.88 & [18], \\ -72.11 \pm 0.27 \pm 0.88 & [19], \end{cases} \tag{3.27}$$

where the first error is experimental while the second one is theoretical. Both theoretical errors come from the same estimation [20], and hence they are 100% correlated. By combining these two data, we obtain

$$Q_W({}^{133}_{55}Cs) = -72.08 \pm 0.92. \tag{3.28}$$

APV experiments in the thallium atom have been performed by Seattle [21] and Oxford [22] groups. The results are;

$$Q_W({}^{205}_{81}Tl) = \begin{cases} -114.2 \pm 1.3 \pm 4.0 & [21], \\ -120.5 \pm 3.5 \pm 4.0 & [22]. \end{cases} \tag{3.29}$$

The result in the second line is obtained in [23] where $Q_W$ was extracted from the report [22] of the Oxford group. In [21], a part of the theoretical error that has been accounted for in [22] was not included. Therefore we replaced the theoretical error of [21] by $\pm 4.0$, following [23]. The two common theoretical errors are again 100% correlated. The combined result is given by

$$Q_W({}^{205}_{81}Tl) = -115.0 \pm 4.2. \tag{3.30}$$

Theoretical predictions for the weak charges of $Cs$ and $Tl$ are

$$Q^{\text{‘SM’}}_W({}^{133}_{55}Cs) \approx -73.07 - 0.745\Delta S - 0.054\Delta T, \tag{3.31a}$$
$$Q^{\text{‘SM’}}_W({}^{205}_{81}Tl) \approx -116.6 - 1.10\Delta S - 0.15\Delta T, \tag{3.31b}$$

in the generic $\text{SU}(2)_L \times \text{U}(1)_Y$ model. In the minimal SM, they are given as;

$$Q^{\text{SM}}_W({}^{133}_{55}Cs) \approx -73.07 - 0.003x_t - 0.062x_H + 0.007x_H^2, \tag{3.32a}$$
$$Q^{\text{SM}}_W({}^{205}_{81}Tl) \approx -116.6 - 0.01x_t - 0.09x_H + 0.01x_H^2. \tag{3.32b}$$

### 3.3 $\nu_\mu$-quark scattering

For neutrino-quark scattering, the experimental results are presented by using the model-independent parameters $g_\alpha^2$ and $\delta_\alpha^2$ (or equivalently $q_\alpha$) of [24]. We examine two independent sets of experimental data; the results of all the old neutrino-nucleus scattering experiments as summarized by Fogli and Haidt (FH) [24] and the recent CCFR measurement [25].

The result of [24] can be expressed as [6]

$$\left.\begin{aligned} g_L^2 &= \phantom{-}0.2980 \pm 0.0044 \\ g_R^2 &= \phantom{-}0.0307 \pm 0.0047 \\ \delta_L^2 &= -0.0589 \pm 0.0237 \\ \delta_R^2 &= \phantom{-}0.0206 \pm 0.0160 \end{aligned}\right\} \quad \rho_{\rm corr} = \begin{pmatrix} 1 & -0.559 & -0.163 & 0.162 \\ & 1 & 0.156 & -0.037 \\ & & 1 & -0.447 \\ & & & 1 \end{pmatrix}. \tag{3.33}$$

Typical momentum transfer of these measurements is $\langle Q^2\rangle_{\rm FH} = 20$ GeV$^2$.

The experiment of CCFR collaboration has been done at slightly higher momentum transfer $\langle Q^2\rangle_{\rm CCFR} = 35$ GeV$^2$. The result was given for the following combination of the model-independent parameters [25];

$$K = 1.7897 g_L^2 + 1.1479 g_R^2 - 0.0916\delta_L^2 - 0.0782\delta_R^2, \tag{3.34a}$$
$$= 0.5820 \pm 0.0049. \tag{3.34b}$$

Taking into account of the difference of the typical momentum transfer of the two data sets, we find the following theoretical predictions for the model-independent parameters;

$$u_L^{\text{‘SM’}} \approx \begin{pmatrix} 0.3465 \\ 0.3468 \end{pmatrix} - 0.0023\Delta S + 0.0041\Delta T, \tag{3.35a}$$
$$u_R^{\text{‘SM’}} \approx \begin{pmatrix} -0.1549 \\ -0.1549 \end{pmatrix} - 0.0023\Delta S + 0.0004\Delta T, \tag{3.35b}$$
$$d_L^{\text{‘SM’}} \approx \begin{pmatrix} -0.4296 \\ -0.4299 \end{pmatrix} + 0.0012\Delta S - 0.0039\Delta T, \tag{3.35c}$$
$$d_R^{\text{‘SM’}} \approx \begin{pmatrix} 0.0776 \\ 0.0775 \end{pmatrix} + 0.0012\Delta S - 0.0002\Delta T, \tag{3.35d}$$

in the generic $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ models, and

$$u_L^{\rm SM} \approx \begin{pmatrix} 0.3465 \\ 0.3468 \end{pmatrix} + 0.0005x_t - 0.00065x_H, \tag{3.36a}$$
$$u_R^{\rm SM} \approx \begin{pmatrix} -0.1549 \\ -0.1549 \end{pmatrix} + 0.0001x_t - 0.00022x_H, \tag{3.36b}$$
$$d_L^{\rm SM} \approx \begin{pmatrix} -0.4296 \\ -0.4299 \end{pmatrix} - 0.0005x_t + 0.00055x_H, \tag{3.36c}$$
$$d_R^{\rm SM} \approx \begin{pmatrix} 0.0776 \\ 0.0775 \end{pmatrix} + 0.00011x_H, \tag{3.36d}$$

in the minimal SM. In the above, the upper and the lower numbers in each column are the predictions at $\langle Q^2\rangle_{\rm FH} = 20$ GeV$^2$ and at $\langle Q^2\rangle_{\rm CCFR} = 35$ GeV$^2$, respectively. The $Q^2$-dependences of the theoretical predictions turned out to be negligibly small.

### 3.4 $\nu_\mu$-electron scattering

There are three experimental results for the $\nu_\mu$-electron scattering [26]. Here we use the combined data which are expressed in terms of the cross sections [6];

$$\left.\begin{aligned} \left[\sigma^{\nu e}/E_\nu\right](10^{-42}\mathrm{cm}^2/\mathrm{GeV}) &= 1.56 \pm 0.10, \\ \left[\sigma^{\overline{\nu} e}/E_{\overline{\nu}}\right](10^{-42}\mathrm{cm}^2/\mathrm{GeV}) &= 1.36 \pm 0.09, \end{aligned}\right\} \quad \rho_{\rm corr} = 0.51. \tag{3.37}$$

As seen from (2.11), the above cross sections expressed in terms of the helicity amplitudes, $M_{L\alpha}^{\nu_\mu e}$. If we ignore the $Q^2$-dependence of their matrix elements, the experimental data (3.37) can be expressed in terms of the two independent helicity amplitudes as

$$\left.\begin{aligned} M_{LL}^{\nu_\mu e} &= \phantom{-}8.87 \pm 0.35, \\ M_{LR}^{\nu_\mu e} &= -7.73 \pm 0.36, \end{aligned}\right\} \quad \rho_{\rm corr} = 0.13, \tag{3.38}$$

in units of TeV$^{-2}$. The theoretical predictions in the generic $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ models are

$$(M_{LL}^{\nu_\mu e})^{\text{‘SM’}} \approx \phantom{-}9.02 - 0.11\Delta S + 0.14\Delta T, \tag{3.39a}$$
$$(M_{LR}^{\nu_\mu e})^{\text{‘SM’}} \approx -7.70 - 0.11\Delta S + 0.02\Delta T, \tag{3.39b}$$

and those within the minimal SM are,

$$(M_{LL}^{\nu_\mu e})^{\rm SM} \approx \phantom{-}9.02 + 0.02x_t - 0.025x_H, \tag{3.40a}$$
$$(M_{LR}^{\nu_\mu e})^{\rm SM} \approx -7.70 + 0.003x_t - 0.011x_H, \tag{3.40b}$$

all in units of TeV$^{-2}$. The $\langle Q^2\rangle$-dependence of the theoretical predictions of the matrix elements $M_{L\alpha}^{\nu_\mu e}(\langle Q^2\rangle)$ has indeed been found to be negligible between $\langle Q^2\rangle = m_e E_\nu$ and $\langle Q^2\rangle = m_e E_\nu/2$ in (2.11), for $E_\nu = 25.7$ GeV (CHARM II [26]).

## 4 Results on generic $\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$ model

### 4.1 Constraints on $S, T$ parameters

Here, we study the constraints on $S$ and $T$ parameters in the generic $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ model from the low-energy electroweak experiments listed in the previous section.

(i) *Asymmetries in $\ell$-$q$ scatterings experiments;*
We combine the results of the four experiments, SLAC ($e$D), CERN ($\mu^\pm$C), Bates ($e$C), Mainz ($e$Be) and find

$$S'' = -3.75 + 0.93T \pm 2.80. \tag{4.1}$$

(ii) *APV experiments;*
The combined result of parity violation experiments in the cesium and thallium atoms is given by

$$S'' = -1.6 - 0.063T \pm 1.2. \tag{4.2}$$

As emphasized in [23,27], the APV experiments constrain mainly the S parameter.

(iii) *$\nu_\mu$-$q$ scatterings;*
From the two data sets of the $\nu_\mu$-$q$ scattering experiments given in Sect. 3.3, the following constraint is obtained,

$$\left.\begin{aligned} S'' &= 0.95 \pm 5.01 \\ T &= 0.75 \pm 1.78 \end{aligned}\right\} \quad \rho_{\rm corr} = 0.979. \tag{4.3}$$

Because of the strong positive correlation between the errors, only the following combination is effectively constrained;

$$T = 0.42 + 0.35S'' \pm 0.36. \tag{4.4}$$

(iv) *$\nu_\mu$-$e$ scatterings;*
From the $\nu_\mu$-$e$ scattering data in Sect. 3.4, we find

$$\left.\begin{aligned} S'' &= 0.035 \pm 3.6 \\ T &= 0.064 \pm 3.9 \end{aligned}\right\} \quad \rho_{\rm corr} = 0.76. \tag{4.5}$$

Summing up all the L.E.N.C. experiments, we find the following constraints on the $S$ and $T$ parameters,

$$\left.\begin{aligned} S'' &= -1.44 \pm 1.03 \\ T &= -0.06 \pm 0.51 \end{aligned}\right\} \quad \rho_{\rm corr} = 0.715, \tag{4.6}$$
$$\chi^2_{\rm min}/({\rm d.o.f}) = 3.38/(13).$$

We show in Fig. 1 our results of the individual constraints, (4.1), (4.2), (4.3) and (4.5) as well as the combined result (4.6) for $\delta_\alpha = 0.03$ [11]. When compared with corresponding result of [9,10], significant improvements are found in the constraints from the APV and the $\nu_\mu$-$q$ scattering experiments. Constraints from the $\ell$-$q$ scattering experiments are only slightly improved by including the CERN $\mu^\pm$C, Bates $e$C and Mainz $e$Be experiments in the analysis.

### 4.2 Comparison with LEP/SLC data

From the updated $Z$ shape parameter measurements by LEP/SLC, the effective charges $\bar{s}^2(m_Z^2)$ and $\bar{g}_Z^2(m_Z^2)$ have been extracted in [9]. It is assumed that the vertex functions beside the $Zb_Lb_L$ vertex function $\bar{\delta}_b(m_Z^2)$ are dominated by the SM contributions and the following combination [6,9]

$$\alpha_s' = \alpha_s(m_Z^2)_{\overline{\rm MS}} + 1.54[\bar{\delta}_b(m_Z^2) + 0.00995] \tag{4.7}$$

is constrained by the hadronic decay width of the $Z$-boson. Then, by taking $\alpha_s'$ and $\bar{\delta}_b(m_Z^2)$ as external parameters, the following result has been found in [9];

$$\left.\begin{aligned} \bar{g}_Z^2(m_Z^2) &= 0.55557 - 0.00042\,\tfrac{\alpha_s'-0.1218}{0.0038} \pm 0.00061 \\ \bar{s}^2(m_Z^2) &= 0.23065 + 0.00003\,\tfrac{\alpha_s'-0.1218}{0.0038} \pm 0.00024 \end{aligned}\right\}$$
$$\rho_{\rm corr} = 0.24, \tag{4.8a}$$

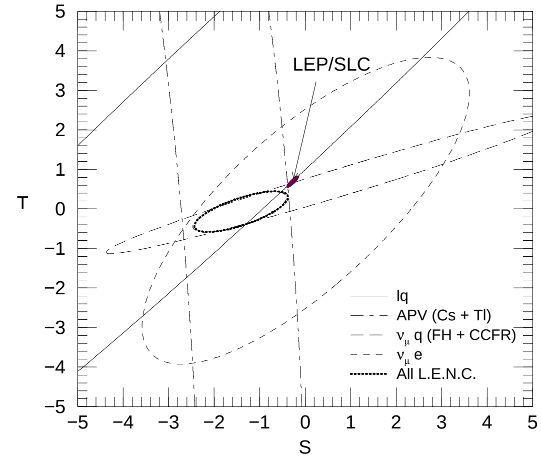


**Fig. 1.** Fit to L.E.N.C. data on the $(S,T)$ plane. The 1-$\sigma$ (39% CL) allowed ranges are shown separately for charged lepton-quark scattering experiments, atomic parity violation, $\nu_\mu$-quark scattering and $\nu_\mu$-$e$ scattering experiments. Also shown are the 1-$\sigma$ allowed region of all the L.E.N.C. experiments as well as that of the LEP/SLC $Z$-pole measurements

$$\chi^2_{\rm min} = 15.4 + \left(\frac{\alpha_s' - 0.1218}{0.0038}\right)^2 + \left(\frac{\bar{\delta}_b + 0.0051}{0.0028}\right)^2. \tag{4.8b}$$

with (d.o.f) = 11. The above result can be expressed in terms of the $S$ and $T$-parameters as follows;

$$\left.\begin{aligned} S' &= -0.29 - 0.059\,\tfrac{\alpha_s'-0.1218}{0.0038} \pm 0.13 \\ T &= \phantom{-}0.69 - 0.102\,\tfrac{\alpha_s'-0.1218}{0.0038} \pm 0.15 \end{aligned}\right\}$$
$$\rho_{\rm corr} = 0.87, \tag{4.9a}$$
$$\chi^2_{\rm min} = 15.4 + \left(\frac{\alpha_s' - 0.1218}{0.0038}\right)^2 + \left(\frac{\bar{\delta}_b + 0.0051}{0.0028}\right)^2. \tag{4.9b}$$

Here the combination $S' = S - 0.72\delta_\alpha$ is determined from the $\bar{s}^2(m_Z^2)$ measurement [9]. It is slightly different from the combination $S''$ of (2.14) which is constrained from the $\bar{s}^2(0)$ measurement at low-energies. The difference coms from the uncertainty in the hadronic corrections to the running of the effective charge $\bar{e}^2(q^2)/\bar{s}^2(q^2)$ between $q^2 = m_Z^2$ and $q^2 = 0$; see Appendix B of [6]. The result is shown in Fig. 1 for $\delta_\alpha = 0.03$ [11] and $\alpha_s' = 0.1218$. We find that the $Z$-pole results (4.9) are consistent with the SM predictions for $x_t = x_H = 0$, $(S,T) = (-0.23, 0.88)$, whereas the results of the L.E.N.C. experiments (4.6) give slightly smaller $S$ and $T$.

We can combine all the L.E.N.C. data with the above LEP/SLC data, and find for $\delta_\alpha = 0.03$,

$$\left.\begin{aligned} S &= -0.33 - 0.051\,\tfrac{\alpha_s'-0.1227}{0.0037} \pm 0.13 \\ T &= \phantom{-}0.61 - 0.089\,\tfrac{\alpha_s'-0.1227}{0.0037} \pm 0.14 \end{aligned}\right\}$$

$$\rho_{\rm corr} = 0.86, \qquad (4.10a)$$

$$\chi^2_{\rm min} = 20.8 + \left(\frac{\alpha_s' - 0.1227}{0.0037}\right)^2 + \left(\frac{\bar{\delta}_b + 0.0051}{0.0028}\right)^2, \qquad (4.10b)$$

with (d.o.f) = (15(L.E.N.C.)+13(LEP/SLC)) − 2 = 26. The combined fit does not change significantly from the previous result of [9] mainly because of the dominant role of the $Z$-pole data.

## 5 Constraints on contact terms

In this section, we study the constraints on the contact interactions from all the L.E.N.C. data.

### 5.1 Constraints on general contact interactions

In the general framework of the contact interactions, each L.E.N.C. experiment constrains certain combinations of the contact terms.

From the SLAC $e$D experiments, two combinations of the model independent parameters $C_{1q}^e$ and $C_{2q}^e$ are constrained. Assuming the SM prediction (3.6), the experimental data (3.4) lead the following constraints;

$$\left.\begin{aligned} \Delta(2C_{1u}^e - C_{1d}^e) &= \phantom{-}0.217 \pm 0.26 \\ \Delta(2C_{2u}^e - C_{2d}^e) &= -0.765 \pm 1.23 \end{aligned}\right\} \rho_{\rm corr} = -0.975. \qquad (5.1)$$

Here and in the following, we adopt the SM predictions for $m_t = 175$ GeV and $m_H = 100$ GeV (and $\delta_\alpha = 0.03$ [11]). Effects of small changes in the SM predictions for different values of $m_t$ and $m_H$ (or for $S$ and $T$) can easily be accounted for by using the parametrizations given in Sect. 3.

The CERN $\mu^\pm$C experiment is used to extract the model independent parameters $C_{2q}^\mu$ and $C_{3q}^\mu$. From the experimental data given in Sect. 3, we find the following constraints on the linear combination of $\Delta C_{2q}^\mu$ and $\Delta C_{3q}^\mu$;

$$\left.\begin{aligned} \Delta(2C_{3u}^\mu - C_{3d}^\mu) &= -1.51 \pm 4.9 \\ \Delta(2C_{2u}^\mu - C_{2d}^\mu) &= \phantom{-}1.74 \pm 6.3 \end{aligned}\right\} \rho_{\rm corr} = -0.997. \qquad (5.2)$$

The Bates experiment on $e$C scattering constrains the following combination;

$$\Delta(C_{1u}^e + C_{1d}^e) = 0.0152 \pm 0.033. \qquad (5.3)$$

The Mainz experiment on $e$Be scattering constrains the combination

$$\begin{aligned} \Delta A_{Mainz} &= -2.73\Delta C_{1u}^e + 0.65\Delta C_{1d}^e - 2.19\Delta C_{2u}^e \\ &\quad +2.03\Delta C_{2d}^e \\ &= -0.065 \pm 0.19. \end{aligned} \qquad (5.4)$$

The two APV experiments give;

$$\begin{aligned} \Delta Q_W(Cs) &= 376\Delta C_{1u}^e + 422\Delta C_{1d}^e \\ &= 0.96 \pm 0.92, \end{aligned} \qquad (5.5)$$

$$\Delta Q_W(Tl) = 572\Delta C_{1u}^e + 658\Delta C_{1d}^e = 1.58 \pm 4.2. \qquad (5.6)$$

By combining the two sets of the $\nu_\mu$-$q$ scattering data, we find

$$\left.\begin{aligned} \Delta u_L &= -0.0032 \pm 0.0169 \\ \Delta u_R &= -0.0084 \pm 0.0251 \\ \Delta d_L &= \phantom{-}0.0020 \pm 0.0136 \\ \Delta d_R &= -0.0109 \pm 0.0631 \end{aligned}\right\}$$

$$\rho_{\rm corr} = \begin{pmatrix} 1 & 0.385 & 0.954 & 0.412 \\ & 1 & 0.355 & 0.841 \\ & & 1 & 0.511 \\ & & & 1 \end{pmatrix}. \qquad (5.7)$$

The contact term contributions to the model-independent parameters $\Delta C_{iq}$ and $\Delta q_\alpha$ are found in (2.7) and (2.9), respectively.

From the $\nu_\mu$-$e$ scattering experiments, (3.37), we find

$$\left.\begin{aligned} \Delta M_{LL}^{\nu_\mu e} &= \eta_{LL}^{\nu_\mu e} = -0.15 \pm 0.35 \\ \Delta M_{LR}^{\nu_\mu e} &= \eta_{LR}^{\nu_\mu e} = -0.03 \pm 0.36 \end{aligned}\right\} \rho_{\rm corr} = 0.13, \qquad (5.8)$$

in units of TeV$^{-2}$. The two pure-leptonic contact interactions are constrained directly by these experiments.

### 5.2 Constraints on $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ invariant contact interactions

Without any further assumptions, there are 108 lepton-quark contact couplings $\eta_{\alpha\beta}^{\ell q}$. In this subsection, we combine the individual constraints (5.1) ∼ (5.7) by taking the following assumptions;

(i) *lepton universality*

$$\eta_{\alpha\beta}^{eq} = \eta_{\alpha\beta}^{\mu q} = \eta_{\alpha\beta}^{\tau q} \equiv \eta_{\alpha\beta}^{\ell q}, \qquad (5.9)$$

(ii) *quark universality*

$$\eta_{\alpha\beta}^{\ell u} = \eta_{\alpha\beta}^{\ell c} = \eta_{\alpha\beta}^{\ell t}, \qquad (5.10a)$$

$$\eta_{\alpha\beta}^{\ell d} = \eta_{\alpha\beta}^{\ell s} = \eta_{\alpha\beta}^{\ell b}, \qquad (5.10b)$$

(iii) $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ *invariance (for* $\mathrm{SU(2)}_L$ *singlet exchange)*

$$\eta_{\alpha L}^{\ell u} = \eta_{\alpha L}^{\ell d}, \qquad (5.11a)$$

$$\eta_{L\beta}^{\nu_\ell q} = \eta_{L\beta}^{\ell q}. \qquad (5.11b)$$

Six lepton-quark contact terms remain as independent parameters;

$$\left\{\eta^{\ell u}_{LL}, \eta^{\ell u}_{LR}, \eta^{\ell d}_{LR}, \eta^{\ell d}_{RR}, \eta^{\ell u}_{RL}, \eta^{\ell u}_{RR}\right\}. \tag{5.12}$$

Let us first examine how the above six contact interaction terms are constrained from each low-energy experiment. We express the model-independent parameters, $\Delta C_{iq}, \Delta q_\alpha$ and $\Delta M^{\ell q}_{\alpha\beta}$ in terms of $\eta^{\ell q}_{\alpha\beta}$ with the above assumptions. We find the following constraints on the various combinations of the contact terms: Here $\eta^{\ell q}_{\alpha\beta}$ is measured in units of TeV$^{-2}$.

SLAC $e$D experiment:

$$\begin{aligned}&-0.5\eta^{\ell u}_{LL} - 0.657\eta^{\ell u}_{LR} + 0.329\eta^{\ell d}_{LR} + 0.329\eta^{\ell u}_{RL}\\ &+\eta^{\ell u}_{RR} - 0.5\eta^{\ell d}_{RR} = -0.86 \pm 0.79.\end{aligned} \tag{5.13}$$

CERN $\mu^\pm$C experiment:

$$\begin{aligned}&0.062\eta^{\ell u}_{LL} - 0.124\eta^{\ell u}_{LR} + 0.062\eta^{\ell d}_{LR} - 0.5\eta^{\ell u}_{RL}\\ &+\eta^{\ell u}_{RR} - 0.5\eta^{\ell d}_{RR} = 1.4 \pm 3.5.\end{aligned} \tag{5.14}$$

Bates $e$C experiment:

$$\begin{aligned}&\eta^{\ell u}_{LL} + 0.5\eta^{\ell u}_{LR} + 0.5\eta^{\ell d}_{LR} - \eta^{\ell u}_{RL}\\ &-0.5\eta^{\ell u}_{RR} - 0.5\eta^{\ell d}_{RR} = 0.25 \pm 0.54.\end{aligned} \tag{5.15}$$

Mainz $e$Be experiment:

$$\begin{aligned}&-0.455\eta^{\ell u}_{LL} - 0.110\eta^{\ell u}_{LR} - 0.280\eta^{\ell d}_{LR} + 0.390\eta^{\ell u}_{RL}\\ &+\eta^{\ell u}_{RR} - 0.545\eta^{\ell d}_{RR} = -0.44 \pm 1.27.\end{aligned} \tag{5.16}$$

APV($Cs$):

$$\begin{aligned}&\eta^{\ell u}_{LL} + 0.471\eta^{\ell u}_{LR} + 0.529\eta^{\ell d}_{LR} - \eta^{\ell u}_{RL}\\ &-0.471\eta^{\ell u}_{RR} - 0.529\eta^{\ell d}_{RR} = 0.040 \pm 0.038.\end{aligned} \tag{5.17}$$

APV($Tl$):

$$\begin{aligned}&\eta^{\ell u}_{LL} + 0.465\eta^{\ell u}_{LR} + 0.535\eta^{\ell d}_{LR} - \eta^{\ell u}_{RL}\\ &-0.465\eta^{\ell u}_{RR} - 0.535\eta^{\ell d}_{RR} = 0.042 \pm 0.113.\end{aligned} \tag{5.18}$$

In (5.13) and (5.14), looser constraints are dropped.

From the result of $\nu_\mu$-$q$ experiments (5.7), the following three contact terms are constrained;

$$\left.\begin{aligned}\eta^{\ell u}_{LL} &= -0.22 \pm 0.40\\ \eta^{\ell u}_{LR} &= \phantom{-}0.06 \pm 0.83\\ \eta^{\ell d}_{LR} &= \phantom{-}0.47 \pm 2.14\end{aligned}\right\},\quad \rho_{\text{corr}} = \begin{pmatrix}1 & 0.27 & 0.54\\ & 1 & 0.89\\ & & 1\end{pmatrix}. \tag{5.19}$$

where, the contact terms are given in units of TeV$^{-2}$.

By adding all data of low-energy lepton-quark experiments given in (5.1) $\sim$ (5.7), we find[2]

$$\left.\begin{aligned}\eta^{\ell u}_{LL} &= -0.281 \pm 0.375\\ \eta^{\ell u}_{LR} &= -0.081 \pm 0.739\\ \eta^{\ell d}_{LR} &= \phantom{-}0.02 \pm 1.43\\ \eta^{\ell u}_{RL} &= -2.47 \pm 4.00\\ \eta^{\ell u}_{RR} &= \phantom{-}1.39 \pm 3.53\\ \eta^{\ell d}_{RR} &= \phantom{-}2.76 \pm 4.43\end{aligned}\right\}, \tag{5.20a}$$

$$\rho_{\text{corr}} = \begin{pmatrix}1 & 0.14 & 0.45 & 0.12 & 0.05 & 0.09\\ & 1 & 0.86 & 0.13 & 0.11 & 0.15\\ & & 1 & 0.17 & 0.10 & 0.17\\ & & & 1 & -0.95 & -0.94\\ & & & & 1 & 0.97\\ & & & & & 1\end{pmatrix}, \tag{5.20b}$$

$$\chi^2_{\text{min}}/(\text{d.o.f}) = 2.8/(7), \tag{5.20c}$$

in units of TeV$^{-2}$.

We give below the three eigenvectors of the $(6 \times 6)$ covariance matrix with the smallest errors:

$$\begin{aligned}&0.353\eta^{\ell u}_{LL} + 0.167\eta^{\ell u}_{LR} + 0.185\eta^{\ell d}_{LR} - 0.351\eta^{\ell u}_{RL}\\ &-0.165\eta^{\ell u}_{RR} - 0.185\eta^{\ell d}_{RR} = 0.014 \pm 0.021\end{aligned} \tag{5.21a}$$

$$\begin{aligned}&0.283\eta^{\ell u}_{LL} + 0.394\eta^{\ell u}_{LR} - 0.382\eta^{\ell d}_{LR} + 0.182\eta^{\ell u}_{RL}\\ &+0.044\eta^{\ell u}_{RR} + 0.129\eta^{\ell d}_{RR} = -0.15 \pm 0.25\end{aligned} \tag{5.21b}$$

$$\begin{aligned}&0.775\eta^{\ell u}_{LL} - 0.726\eta^{\ell u}_{LR} + 0.009\eta^{\ell d}_{LR} + 0.288\eta^{\ell u}_{RL}\\ &+0.270\eta^{\ell u}_{RR} + 0.047\eta^{\ell d}_{RR} = -0.36 \pm 0.45\ .\end{aligned} \tag{5.21c}$$

The most accurate constraint (5.21a) is essentially follows from the APV measurements (5.17) and (5.18). The second most accurate constraint (5.21b) is essentially obtained by the $\nu_\mu$-$q$ scattering data (5.7) or (5.19).

Finally the $\nu_\mu$-$e$ scattering experiments constrain the pure-leptonic contact interactions $\eta^{\ell\ell}_{\alpha\beta}$. We find in units of TeV$^{-2}$;

$$\left.\begin{aligned}\eta^{\ell\ell}_{LL} &= -0.15 \pm 0.35\\ \eta^{\ell\ell}_{LR} &= -0.03 \pm 0.36\end{aligned}\right\}\quad \rho_{\text{corr}} = 0.13. \tag{5.22}$$

As a reference, the minimal SM (all $\eta^{ff'}_{\alpha\beta} = 0$) gives an excellent fit to all the data

$$\chi^2_{\text{SM}}/(\text{d.o.f}) = 6.9/(15). \tag{5.23}$$

Therefore, low-energy electroweak experiments do not require any new interactions.

[2] In the actual fit, we used the original data, (3.33) and (3.34), instead of (5.7) to retain accuracy. This explains that the d.o.f. in (5.20c) is not 6 but 7. Equation (5.7) should be read as approximate constraints since the $\nu_\mu$-$q$ 'data' expressed in terms of $u_\alpha$ and $d_\alpha$ slightly deviate from the Gaussian

**Table 1.** The hypercharge $Y$ and the extra $\mathrm{U}(1)_E$ charge $Q_E$ of the left-handed quarks and leptons in $Z_\chi, Z_\psi, Z_\eta$ and $Z_\nu$ models

| field | $Y$ | $\sqrt{24}Q_\chi$ | $\sqrt{72/5}Q_\psi$ | $Q_\eta$ | $Q_\nu$ |
|---|---|---|---|---|---|
| $\nu, e$ | $-\frac{1}{2}$ | $+3$ | $+1$ | $+\frac{1}{6}$ | $+\sqrt{\frac{1}{6}}$ |
| $\nu^c$ | $0$ | $-5$ | $+1$ | $-\frac{5}{6}$ | $0$ |
| $e^c$ | $+1$ | $-1$ | $+1$ | $-\frac{1}{3}$ | $+\sqrt{\frac{1}{24}}$ |
| $u, d$ | $+\frac{1}{6}$ | $-1$ | $+1$ | $-\frac{1}{3}$ | $+\sqrt{\frac{1}{24}}$ |
| $u^c$ | $-\frac{2}{3}$ | $-1$ | $+1$ | $-\frac{1}{3}$ | $+\sqrt{\frac{1}{24}}$ |
| $d^c$ | $+\frac{1}{3}$ | $+3$ | $+1$ | $+\frac{1}{6}$ | $+\sqrt{\frac{1}{6}}$ |

## 6 Discussion

In this paper, we have studied the constraints on the four-Fermi contact interactions from low-energy electroweak experiments. From the polarization/charge asymmetry measurement of charged lepton-nucleus scattering experiments, atomic parity violation experiments, and from neutrino-quark scattering experiments, constraints on the lepton-quark contact interactions were obtained, while neutrino-electron scattering experiments constrain the lepton-lepton contact interactions. By assuming the flavor universality and the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge invariance of the contact interactions, those constraints are parametrized conveniently as the constraint (5.20c) for the six lepton-quark contact terms and (5.22) for the two pure-leptonic contact terms.

By using our result, it is easy to examine consequences of models of new physics that affect low-energy electroweak observables. As an example, we briefly study constraints on an extra $Z$-boson in $\mathrm{E}_6$ models. The models contain two additional new neutral gauge bosons, one is the SO(10) singlet $Z_\psi$, and the other one is the SU(5) singlet $Z_\chi$. In general the two gauge bosons are mixed, and the lighter one $Z_E$ is given by the following linear combination;

$$Z_E = Z_\chi \cos\beta_E + Z_\psi \sin\beta_E. \qquad (6.1)$$

Mixing angle $\beta_E = 0, \pi/2, \tan^{-1}(-\sqrt{5/3})$ and $\tan^{-1}(\sqrt{15})$ correspond to $Z_\chi, Z_\psi, Z_\eta$ and $Z_\nu$ models, respectively. Following the Particle Data Group notation [2], the hypercharge $Y$ and the extra $\mathrm{U}(1)_E$ charge $Q_E$ of the left-handed quarks and leptons are given by Table 1. Then, expressing the contact terms by $\mathrm{U}(1)_E$ gauge coupling $g_E$ and the extra gauge boson mass $m_{Z_E}$ as,

$$\eta_{\alpha\beta}^{ff'} = -\frac{g_\alpha^f g_\beta^{f'}}{m_{Z_E}^2}, \qquad (6.2a)$$

$$g_L^f = g_E Q_E^f, \qquad (6.2b)$$

$$g_R^f = -g_E Q_E^{f^C}, \qquad (6.2c)$$

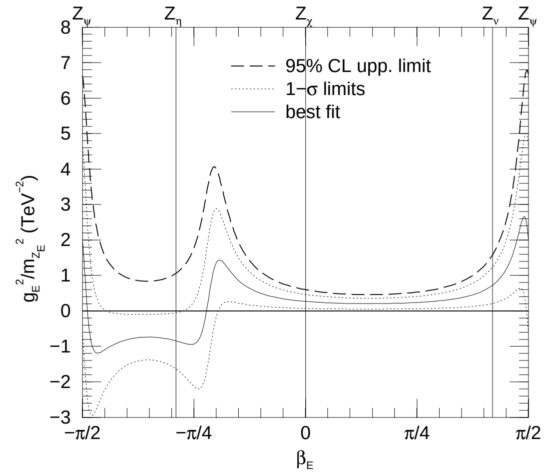


**Fig. 2.** Constraints on $g_E^2/m_{Z_E}^2$ from the low-energy electroweak measurements. The 1-$\sigma$ allowed region and the 95% CL upper bounds are given for the $Z_E$ models that are characterized by the mixing angle $\beta_E$; see (6.1)

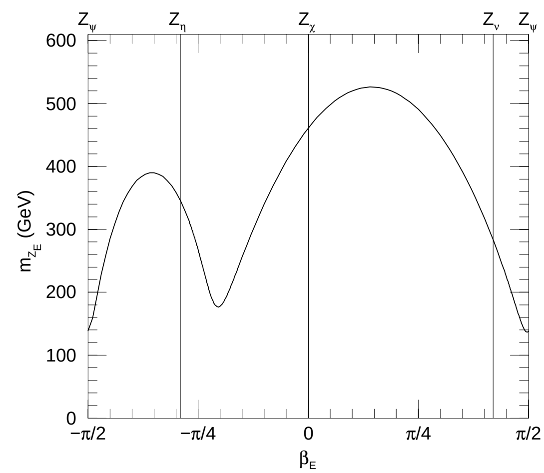


**Fig. 3.** 95% CL lower limits of the extra $Z$ boson mass $m_{Z_E}$ as a function of the mixing angle $\beta_E$

we can find the constraints on each extra $Z$-boson model. We show in Fig. 2 the constraints on $g_E^2/m_{Z_E}^2$ for each model parametrized by the mixing angle $\beta_E$. In the figure, the region of $g_E^2/m_{Z_E}^2 < 0$ is unphysical. The 95% CL upper bounds are obtained under the constraint $g_E^2/m_{Z_E}^2 > 0$. The reason for the appearance of the peak at $|\beta_E| = \pi/2$ ($Z_\psi$ model) can be easily understood as follows; in the $Z_\psi$ model, all left-handed fermions have the same $\mathrm{U}(1)_E$ charge, and hence the couplings are Parity conserving, which makes the most rigorous constraints from APV experiments useless. The set of six couplings, $\eta_{\alpha\beta}^{\ell q}$ , is also less constrained by APV measurements at $\beta_E \simeq \pi/5$, thus the another peak is appeared. To estimate the limit on the extra $Z$-boson mass, the extra $\mathrm{U}(1)_E$ coupling $g_E$ should be fixed. Assuming $g_E^2 = g_Y^2 = \bar{e}^2(m_Z^2)/\bar{c}^2(m_Z^2) = 0.1270$, we show the 95% CL lower limit on the extra $Z$-boson mass in Fig. 3.

**Table 2.** Constraints on $g_E^2/m_{Z_E}^2$ and $m_{Z_E}$ from the low-energy electroweak experiments for the four representative extra $Z$-boson models. We assumed $g_E^2 = g_Y^2 = 0.1270$ to obtain the 95% CL lower limits of $m_{Z_E}$.

| $Z_E$ | $\beta_E$ | $g_E^2/m_{Z_E}^2$ ($\mathrm{TeV}^{-2}$) | $\chi^2_{\min}$ | 95% CL limits | |
|---|---|---|---|---|---|
| | | | | $g_E^2/m_{Z_E}^2$ | $m_{Z_E}$ |
| $Z_\chi$ | $0$ | $0.25 \pm 0.20$ | 5.4 | 0.582 $\mathrm{TeV}^{-2}$ | 470 GeV |
| $Z_\psi$ | $\pi/2$ | $2.0 \pm 2.5$ | 6.4 | 6.44 $\mathrm{TeV}^{-2}$ | 140 GeV |
| $Z_\eta$ | $\tan^{-1}(-\sqrt{5/3})$ | $-0.80 \pm 0.79$ | 6.9 | 1.08 $\mathrm{TeV}^{-2}$ | 340 GeV |
| $Z_\nu$ | $\tan^{-1}(\sqrt{15})$ | $0.69 \pm 0.51$ | 5.2 | 1.54 $\mathrm{TeV}^{-2}$ | 290 GeV |

Constraints on $g_E^2/m_{Z_E}^2$ from the low-energy electroweak experiments for the four representative extra $Z$-boson models are listed in Table 2. The 95% CL upper limits of $g_E^2/m_{Z_E}^2$ and the 95% CL lower limits of $m_{Z_E}$ for $g_E^2 = g_Y^2 = 0.1270$ are also given in the Table 2. As compared to [28], $Z_\chi$ and $Z_\eta$ models are more severely constrained by updated low-energy electroweak experiments.

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